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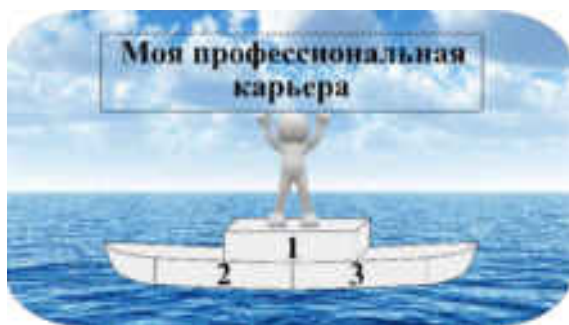


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**Международный научно-образовательный электронный журнал
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Название публикации: «GAUSS AND CRAMER'S METHOD FOR SOLVING LINEAR EQUATIONS»

ABSTRACT

Solving systems of linear equations is a fundamental problem in mathematics, engineering, and computational sciences. Among the most important classical techniques for finding the solutions to such systems are Gauss's method (commonly known as *Gaussian elimination*) and Cramer's method (based on *Cramer's rule*). Both methods provide systematic procedures for determining unknown variables in a set of simultaneous linear equations, but they differ significantly in their computational approach, efficiency, and practical applicability.

Gaussian elimination is an algorithmic method that transforms a system of linear equations into an equivalent upper triangular form through a series of elementary row operations: row swapping, scaling, and row addition. Once the system is in triangular form, the unknown variables can be found using the process of *back substitution*. This method is particularly advantageous for large systems because it can be easily implemented in computer algorithms and extended to matrix decomposition techniques such as *LU decomposition*. Gaussian elimination serves as the foundation for many numerical linear algebra methods and is widely used in scientific computing, data analysis, and engineering modeling.

In contrast, ***Cramer's rule*** provides an explicit formula for each variable in a system of n linear equations with n unknowns using determinants. For a system $AX=B$, where A is a square coefficient matrix and B is the constant vector, the value of each unknown x_i is given by

$$x_i = \frac{\det(A_i)}{\det(A)},$$

where A_i is the matrix obtained by replacing the i -th column of A with the vector B . Cramer's rule is elegant and straightforward for small systems (typically two or three equations) and provides theoretical insights into the relationship between determinants and linear systems. However, it becomes computationally inefficient for large systems because the calculation of determinants involves a large number of arithmetic operations—especially for matrices larger than 3×3 . Moreover, the method requires that $\det(A) \neq 0$, meaning the coefficient matrix must be non-singular.

In practical applications, **Gaussian elimination** is the preferred technique due to its scalability, numerical stability (with pivoting strategies), and suitability for implementation in computer algorithms. On the other hand, **Cramer's rule** is mainly used in theoretical contexts, for symbolic manipulation, or for verifying results in small systems. Both methods, however, play essential roles in linear algebra education, as they illustrate key concepts such as matrix operations, determinants, and the nature of linear dependence among equations.

In summary, Gauss's and Cramer's methods represent two classic yet distinct approaches to solving linear systems. Gauss's method emphasizes computational efficiency and algorithmic structure, while Cramer's rule highlights the elegance of determinant-based solutions. Together, they provide complementary perspectives that bridge the gap between theoretical mathematics and practical computation, forming a cornerstone of modern linear algebra and numerical analysis.

Keywords

Gaussian elimination, Cramer's rule, linear equations, matrix algebra, determinants, LU decomposition, numerical methods, linear systems, back substitution, computational mathematics, matrix operations, numerical analysis.

Introduction

The solution of systems of linear equations is one of the most fundamental problems in mathematics and its applications. Linear equations appear in almost every branch of science and engineering—from modeling electrical circuits, mechanical systems, and

chemical reactions to solving optimization problems, data fitting, and computer graphics. Understanding methods for solving such equations efficiently and accurately is essential for both theoretical mathematics and computational practice. Two classical techniques that have played a crucial role in the history and development of linear algebra are **Gauss's method** (commonly known as *Gaussian elimination*) and **Cramer's method** (known as *Cramer's rule*). Both approaches aim to find the values of unknown variables that satisfy a given set of simultaneous linear equations, but they differ in procedure, computational efficiency, and areas of application.

Background and Importance of Linear Equations

A *linear equation* is an equation that represents a straight line in a coordinate system and involves one or more unknown variables raised only to the first power. When multiple linear equations are considered together, they form what is known as a *system of linear equations*. In matrix form, a system of n equations with n unknowns can be expressed as:

$$AX=B,$$

where A is an $n \times n$ matrix of coefficients, X is a column vector of unknowns, and B is a column vector of constants. The goal of solving this system is to determine the vector X that satisfies the equation.

Systems of linear equations arise naturally in various real-world contexts. For example, in electrical engineering, Kirchhoff's laws yield systems of equations that describe current and voltage distribution in circuits. In structural engineering, equilibrium conditions produce linear systems that determine forces in trusses and beams. In economics, models of supply and demand, input-output analysis, and optimization problems often rely on solving large linear systems. Because of this wide applicability, mathematicians have long sought systematic and efficient methods to obtain solutions.

The history of solving linear systems dates back to ancient times. The earliest known methods can be traced to the Chinese text *The Nine Chapters on the Mathematical Art* (circa 200 BCE), which describes a technique equivalent to modern-day Gaussian elimination for solving simultaneous equations using counting rods.

Centuries later, the Swiss mathematician **Gabriel Cramer** published *Cramer's Rule* in 1750, providing a determinant-based formula for the explicit solution of a system of n linear equations with n unknowns. Later, in the 19th century, **Carl Friedrich Gauss** formalized and popularized a systematic procedure for solving linear systems through elimination, which later became known as *Gaussian elimination*. These developments laid the groundwork for modern matrix theory and computational linear algebra.

Gauss's Method (Gaussian Elimination)

Gaussian elimination is an algorithmic procedure that transforms a given system of equations into an equivalent system that is easier to solve. The process involves a series of *elementary row operations* applied to the augmented matrix $[A|B]$. These operations include:

1. Swapping two rows.
2. Multiplying a row by a nonzero scalar.
3. Adding a multiple of one row to another row.

The objective is to convert the matrix into *row-echelon form* or *reduced row-echelon form*, where the system becomes triangular. Once this is achieved, the unknowns can be found through *back substitution*—starting from the last equation and working upwards to determine the remaining variables.

Gaussian elimination is valued for its systematic nature and computational efficiency. It forms the foundation for many advanced numerical methods, such as *LU decomposition*, *Gauss–Jordan elimination*, and *Gauss–Seidel iteration*. Its algorithmic structure makes it suitable for implementation in computer software, allowing it to handle systems with thousands or even millions of equations—something that would be impossible using purely algebraic methods like Cramer's rule.

However, Gaussian elimination is not without challenges. Numerical instability can occur when small pivot elements lead to large rounding errors during computation. To mitigate this, strategies such as *partial pivoting* or *complete pivoting* are employed. Despite these concerns, Gaussian elimination remains one of the most widely used and reliable techniques for solving linear systems in numerical analysis.

Cramer's Method (Cramer's Rule)

Cramer's rule provides an elegant, determinant-based solution to linear systems. For a system of n linear equations in n unknowns, the rule states that if the determinant of the coefficient matrix A is nonzero, the system has a unique solution given by:

$$x_i = \frac{\det(A_i)}{\det(A)}, i = 1, 2, \dots, n,$$

where A_i is the matrix obtained by replacing the i -th column of A with the constants vector B .

Cramer's rule is notable for its theoretical significance rather than its computational practicality. It provides a direct connection between determinants and systems of equations, offering insight into the existence and uniqueness of solutions. Specifically, if $\det(A)=0$, the system either has no solution or infinitely many solutions.

In practical computation, however, Cramer's rule becomes inefficient as the number of variables increases. The calculation of determinants for large matrices is computationally expensive, involving factorial growth in operations. Therefore, Cramer's rule is primarily used for small systems (such as 2×2 or 3×3) or for symbolic manipulation in theoretical analysis.

Conclusion

The methods developed by Gauss and Cramer represent two of the most influential approaches in the study of linear algebra and the solution of systems of linear equations. Both techniques aim to determine the values of unknown variables that satisfy a given set of equations, yet they differ fundamentally in their methodology and scope of application. ***Gaussian elimination*** provides a systematic, step-by-step algorithm that efficiently reduces equations to a simpler form suitable for back substitution. It is computationally practical and adaptable to large systems, forming the foundation for modern numerical techniques and computer algorithms used in engineering, data analysis, and scientific computation.

Cramer's rule, on the other hand, offers a more theoretical approach based on determinants, illustrating the deep connection between matrix properties and the existence of unique solutions. While elegant and conceptually clear, it becomes

computationally inefficient for systems with many variables, restricting its use to smaller or symbolic problems.

Despite their differences, both methods remain essential in mathematical education and research. Gaussian elimination emphasizes the importance of algorithmic thinking and computational efficiency, while Cramer's rule highlights the structural beauty and theoretical foundations of linear algebra. Together, they demonstrate how classical mathematical methods continue to influence modern problem-solving strategies. Understanding both approaches not only equips students and researchers with essential analytical tools but also fosters a deeper appreciation for the balance between mathematical theory and computational practice.

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